

# Finite Element Supersonic Aerodynamics for Oscillating Parallel Wings

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## Theme

AN analytical development of unsteady supersonic aerodynamic influence coefficients (AIC's) for isolated and nearly parallel interfering coplanar and noncoplanar wings is reported,<sup>1,2</sup> based on triangular discretizations of wings and diaphragms.

In supersonic flow, numerical integration procedures for oscillating wings have been formerly based on the representation of domains of dependence by discrete "elements" including square boxes with sides parallel and normal to the freestream, characteristic boxes with sides parallel to the Mach lines and rectangular ("Mach") boxes with diagonals parallel to the Mach lines. In these approaches, the planform and associated diaphragm boundaries cannot be well represented and constant downwash is usually assumed within an element.

Appa and Smith<sup>3,4</sup> have discussed the use of triangular elements and kinematically consistent integration procedures with the following advantages over the methods referred to above: 1) less dependence of the wing grid on Mach number, 2) continuous distribution of downwash terms across element boundaries, 3) well-represented wing and diaphragm boundaries, and 4) flexibility in the choice of diaphragm elements.

Ashley<sup>5</sup> described concepts of Mach box superposition

methods for application to interfering surfaces which are used in this work. Analytical developments and computer applications using the triangular element for isolated and interfering nearly parallel coplanar and noncoplanar wings at supersonic Mach numbers have been undertaken. Results are compared with published data from other methods.<sup>6-11</sup> Flutter examples of isolated and interfering wings are presented.

Operational computer programs have been used on IBM 360/65 and CDC6600.

## Contents

**A. Isolated Wings:** 1) Table 1 compares generalized aerodynamic coefficients for the "AGARD Swept Wing" in four modes. The comparative number of centerline elements should be noted.<sup>6,7</sup> 2) Figure 1 shows the flutter solution for a swept wing of aspect ratio 2.5 and taper ratio of 0.3.<sup>9</sup>

**B. Interfering Wings:** 1) Table 2 shows generalized aerodynamic damping coefficients at  $M_o = 1.44$  for four modes of the delta wing tail combination of Fig. 2.<sup>10,11</sup> 2) Flutter of interfering NASA Wings at  $M_o = 1.45$  and 1.60 is illustrated in Fig. 3. The wings are of identical planform with zero stagger, aspect ratio 6.8, and taper ratio 0.364. The leading edge sweep is  $16^\circ$ .

**Table 1 Comparison of aerodynamic generalized coefficients  $Q_{ij}$  for AGARD swept wing of aspect ratio 1.45**

$M_o = 1.2, k = 0.5$							
Modes: $q_1 = 1.0, q_2 = (x - c_r/2), q_3 = (x - c_r/2)^2, q_4 = y^2$							
Method	Present		Mach box <sup>6</sup>		Char. box <sup>7</sup>		
No. of $\epsilon$ elements	6		17		30+		
$Q_{ij}$	Real	Imaginary	Real	Imaginary	Real	Imaginary	
$Q_{11}$	-0.0862	3.10	0.0119	3.473	-0.0228	3.3506	
$Q_{12}$	3.46	2.18	3.801	1.681	3.671	1.7325	
$Q_{13}$	3.6	-1.69	3.532	-1.185	3.353	-1.9538	
$Q_{14}$	0.0357	0.766	-0.0894	0.702	0.0319	0.7478	
$Q_{21}$	-0.32	0.13	-0.2695	-0.016	-0.2832	0.0083	
$Q_{22}$	0.37	2.53	0.227	2.475	0.244	2.459	
$Q_{23}$	2.58	-1.11	2.852	-1.055	2.703	-1.507	
$Q_{24}$	-0.0359	0.108	-0.0983	0.022	-0.0216	0.0764	
$Q_{31}$	-0.194	0.5525	-0.1474	0.589	-0.164	0.5532	
$Q_{32}$	0.68	1.45	0.7242	1.276	0.6824	1.305	
$Q_{33}$	1.4	-0.312	1.444	-0.226	1.357	-0.2489	
$Q_{34}$	0.0272	0.0985	-0.0692	0.064	-0.0182	0.0827	
$Q_{41}$	-0.1135	0.723	-0.3301	-0.388	-0.0223	0.8380	
$Q_{42}$	0.82	1.00	-0.3376	1.627	0.9066	0.574	
$Q_{43}$	1.1	-0.386	0.6071	-0.151	0.9925	-0.1059	
$Q_{44}$	-0.0432	0.232	-0.0978	-0.111	-0.018	0.2528	

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Index categories: Supersonic and Hypersonic Flow; Aeroelasticity and Hydroelasticity; Flutter.

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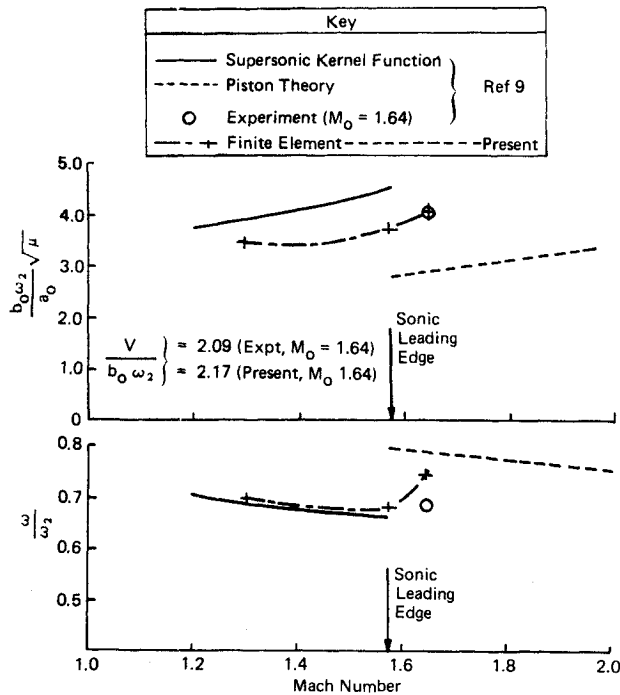


Fig. 1 Flutter boundary in terms of the stiffness-altitude parameter and ratio of flutter frequency to the second natural frequency vs  $M_o$  for HT-7 wing.

Table 2 Comparison of imaginary part of generalized damping coefficients  $Q_{ij}$  for delta wing combination at  $M_o = 1.44$ ,  $k = 0.01$

Modes 1—Heave of wing and tail 2—Pitch of wing and tail		3—Heave, tail only 4—Pitch, tail only	
Method	Present	Ref. 10	Ref. 11
Total no. of elements	Wing 49 tail 16	Wing 300 tail 50	—
Tail $z$ (Fig. 2)			
Imaginary $Q_{ij}$	0.5	0.5	0.5
$Q_{11}$	2.6466	2.954	2.680
$Q_{12}$	0.6670	0.170	0.096
$Q_{13}$	0.3695	0.477	0.393
$Q_{14}$	0.2850	0.387	0.291
$Q_{21}$	0.4280	0.171	0.209
$Q_{22}$	1.1740	0.934	0.598
$Q_{23}$	0.3741	0.483	0.336
$Q_{24}$	0.2920	0.395	0.251
$Q_{31}$	0.4020	0.148	0.157
$Q_{32}$	0.6667	0.343	0.275
$Q_{33}$	0.3696	0.477	0.393
$Q_{34}$	0.2850	0.387	0.291
$Q_{41}$	0.388	0.155	0.141
$Q_{42}$	0.664	0.358	0.245
$Q_{43}$	0.374	0.483	0.336
$Q_{44}$	0.292	0.395	0.251

#### References

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- <sup>2</sup>Paine, A. A., "Development and Applications of Supersonic Unsteady Consistent Aerodynamics for Interfering Parallel

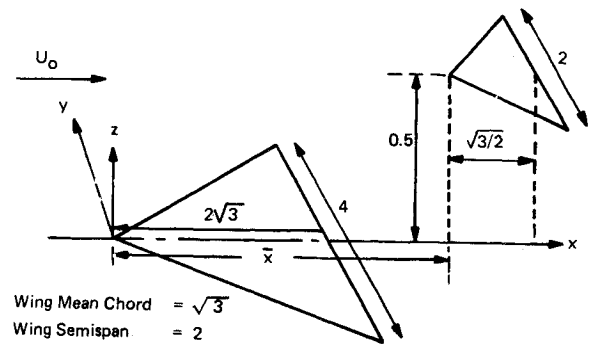


Fig. 2 Configuration of triangular wing—tailplane.<sup>10</sup>

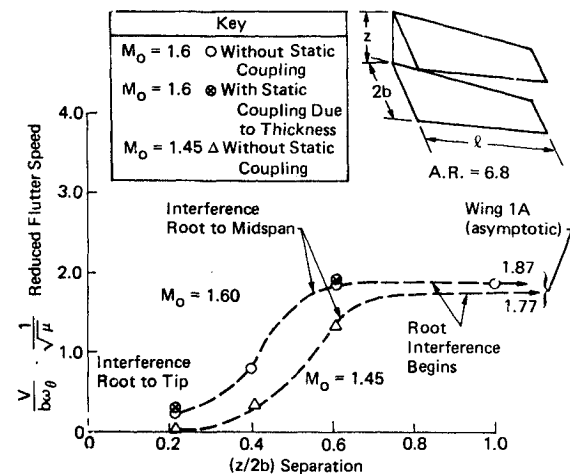


Fig. 3 Variation of reduced flutter speed with vertical separation at  $M_o = 1.45$  and  $M_o = 1.6$ .

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